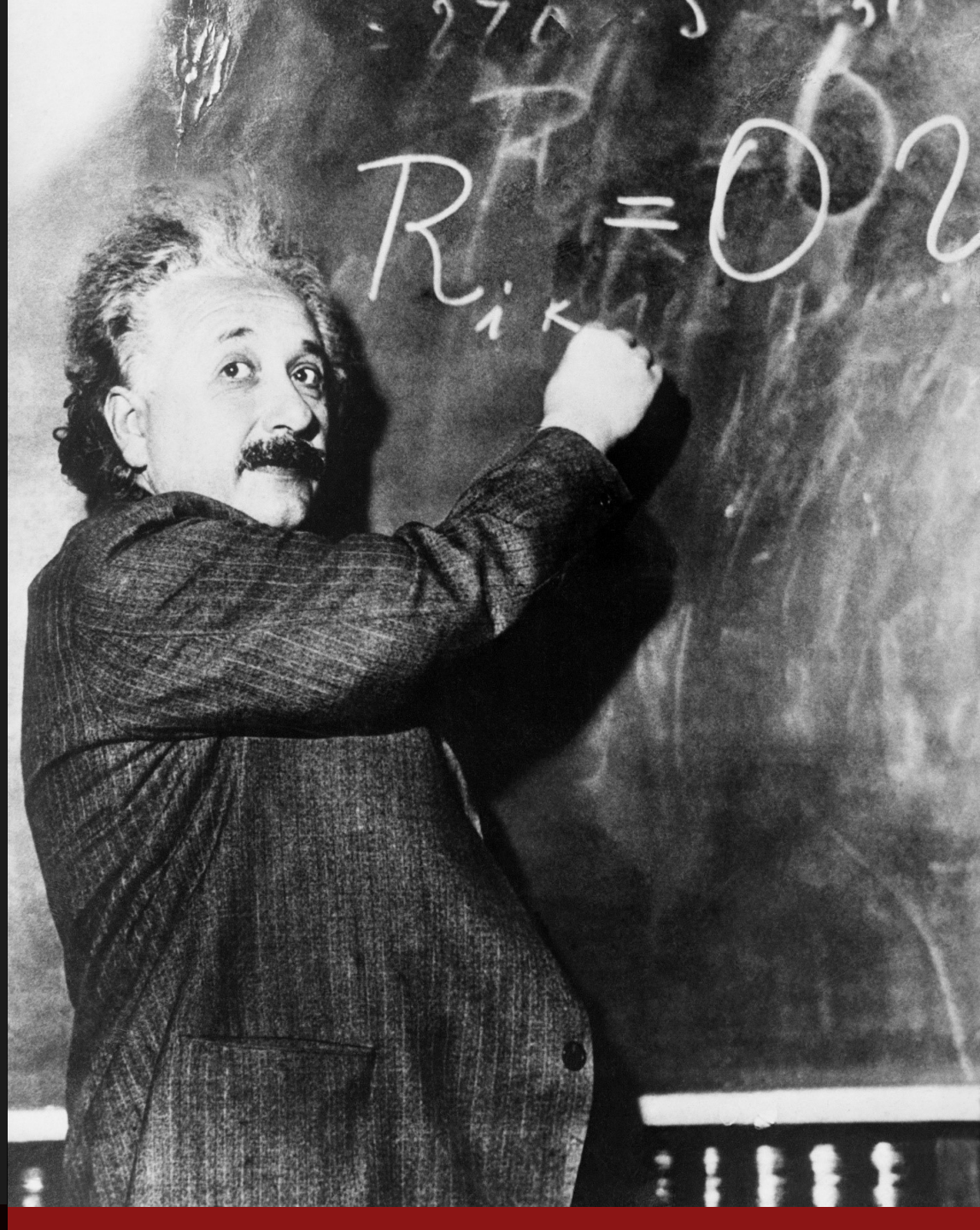
5: Einstein’s Dreams

Physics 17: Black Holes and Extreme Astrophysics



Goals

- Introduce Einstein's theory of special relativity
- Show how the constancy of the speed of light leads to a radical re-interpretation of space and time
- Derive simple formulae that we can use to estimate the physical effects

Newton's Universe

- Space and time are absolute
  - Everyone, everywhere in the Universe, experiences the same passage of time (a second is a second to everyone)
  - Positions and lengths are fixed and space is the 'empty stage' for the Universe (everyone will agree on the length of an object)
  - If one person sees two events as being simultaneous, everyone will see them being simultaneous
- These are approximations that are good so long as we are considering motion at speeds much less than the speed of light ($v \ll c$)

- "Absolute true, and mathematical time, of itself and from its own nature flows equably without relation to anything external…"
- "Absolute space, in its own nature, without relation to anything external, remains always similar and immovable…"

Principa, I. Newton (1687)

The Speed of Light

- Light travels fast, but it does not travel instantly

The speed of light:
$c = 3\times10^{8}\ \mathrm{m\ s^{-1}}$
= 187,000 miles/sec

- When we are looking at distant objects in the Universe, we see them not as they are now, but as they were when the light left (e.g. we see something 1 million ly away as it was 1 million years ago)

Jupiter's 4 Galilean moons are seen to orbit and occasionally disappear behind the planet. Can predict when Io should disappear and re-emerge

As Earth & Jupiter travel around their orbits about the Sun, it was found (Rømer 1676):

- When the Earth & Jupiter are getting closer, Io emerges early (light from 2nd emergence doesn't have to travel as far as light from 1st emergence)
- When the Earth & Jupiter are moving apart, Io emerges late

This can be understood if light travels at a finite speed and takes time (22 min) to get from Jupiter to Earth after Io emerges

Q1:

I'm standing still. I launch a ball forward at 100mph.

You are standing next to me.

What do you measure the ball's speed to be?

Q2:

I'm driving down I-280 at 65mph. I launch a ball forwards out of my car at 100mph.

You are standing by the side of the road.

What do you measure the ball's speed to be?

Now I speed up to 100mph and launch another ball backwards at 100mph. What speed do you measure?

Q3:

I'm riding down I-280 in a car at 65mph. I throw a ball straight up at 10mph and catch it again when it falls back down.

You are standing by the side of the road.

What path do you see the ball follow?

Q4:

I'm driving down I-280 at 65mph with my headlamps on. The light leaves my headlamps at 187,000mps

You are able to measure the speed of the light coming out of my headlamps from the side of the road.

What do you measure the speed of the light to be?

What if instead of my car, it was the light coming out of an airplane flying at 800mph?

When we're both at rest, we measure the speed of the ball to be the same (100mph in each case)

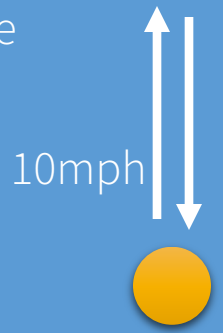
When the ball is launched forwards at 100mph from a car travelling at 65mph, an observer at the side of the road sees the ball travelling forwards at 165mph

$$u' = u + v$$

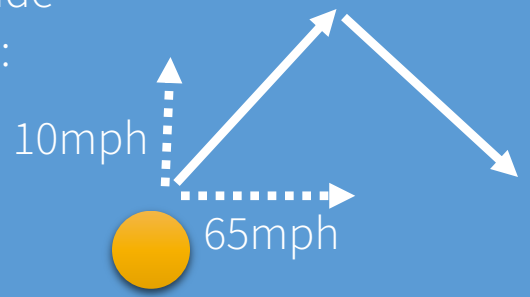
When the ball is launched backwards, the observer sees the ball travelling backward at 100-65 = 35mph

In the car, the velocity of the ball starts at 10mph upwards
From the side of the road, the velocity of the ball starts at 10mph upwards plus 65mph along the road (due to the motion of the car) – the ball goes up & down along a triangular path

Inside car:



From side of road:



The speed of light is constant for all observers.

The driver and the pilot of the plane sees the light leaving the headlamps at 187,000mps

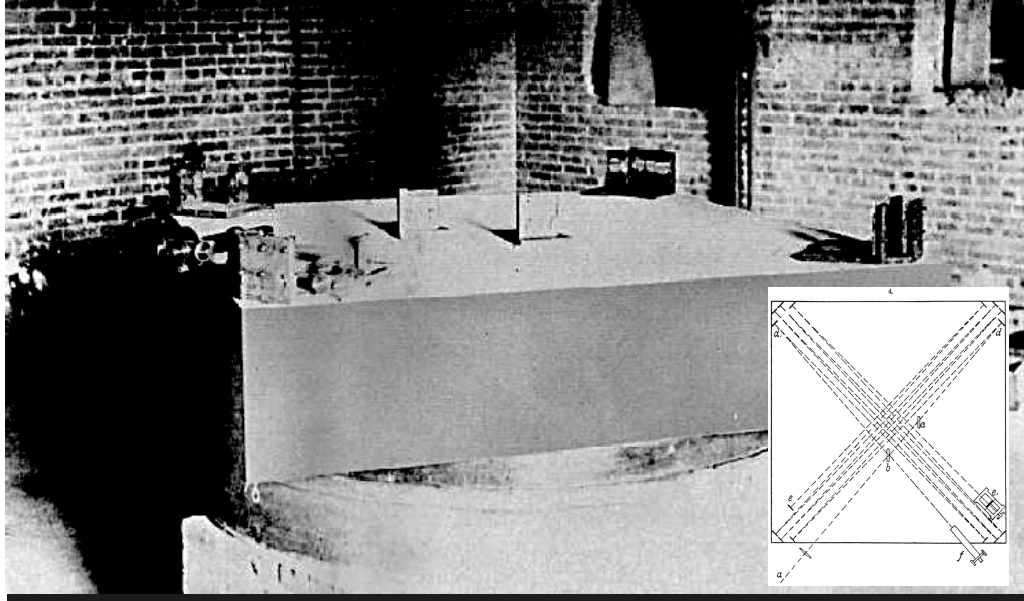
An observer on the ground also sees the light travelling at 187,000mps (the speed of the car/plane is not added to the speed of light)

This is a profound revelation that will lead us to re-interpret space and time!

WILtY

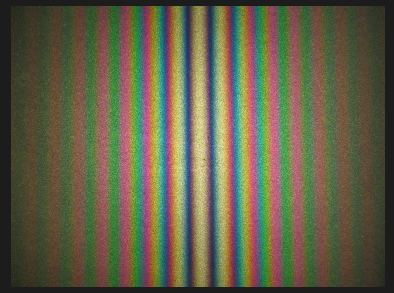
The Speed of Light Revisited

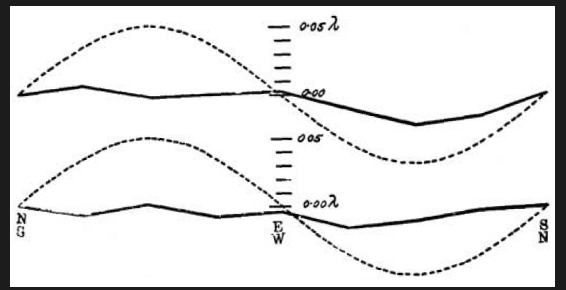
- If light is a wave, it was thought that the wave needed a medium to propagate through (the *luminiferous aether*)
  - The Earth should move through the stationary aether as it orbits the Sun (at ~30km s^{-1})
  - The speed of light should be different parallel and perpendicular to the direction of motion (light should be faster when travelling into the aether, slower when travelling with the aether)
- The Michelson-Morley experiment (1887) measured the relative speed of light in two perpendicular directions
  - Experiment rotated to try different directions
  - Repeated at different times of year as Earth travels around Sun
- Found no difference in speed of light
  - Expected deviation of 0.4 of a fringe, measured maximum 0.02, average less then 0.01, and not at the correct angles
  - The Earth does not travel through a stationary aether



Michelson-Morley Experiment split light into 2 perpendicular paths of equal length, then recombined the beams

The light interferes, producing fringes. If the paths are they same, and light travels each at the same speed, the light waves return in phase (the wave peaks are aligned) so produce a bright fringe in the middle.





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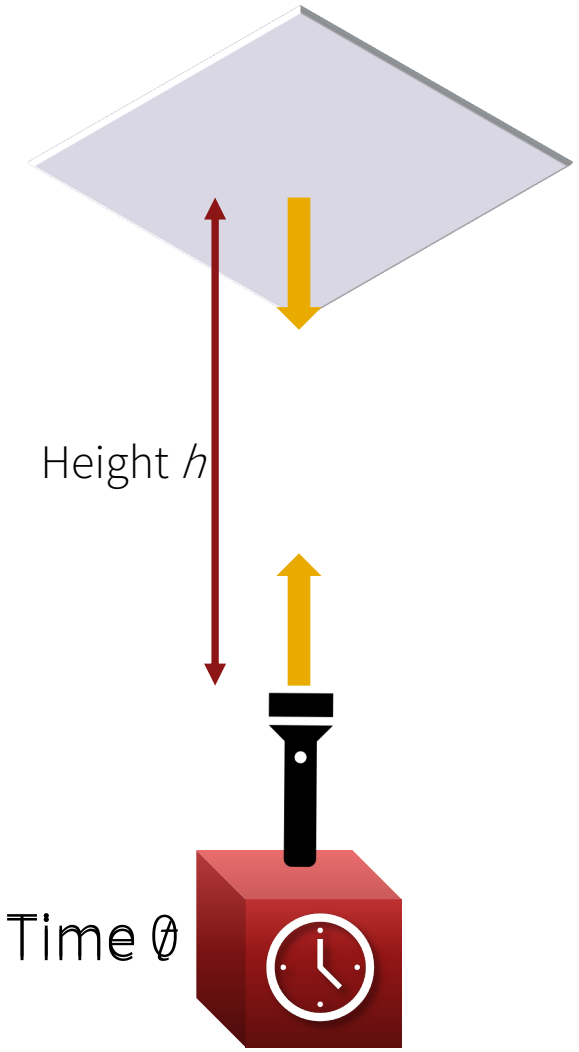
The Principle of Relativity

- Galileo: the laws of motion are the same in all inertial frames. Without an external frame of reference, you cannot sense the motion of your frame (at a constant speed)
  - Even when you're moving, you throw a ball up & down and as far as you see, it travels in a straight line up and down
- *An inertial frame is one either at rest or moving at a constant velocity*

Einstein's central principles of special relativity:

- The laws of physics are the same in all inertial frames
- The speed of light is constant for all observers

What happens if the speed of light is constant?



- Send a flash of light up to a mirror a distance y away at time 0
- Time how long it takes to come back (at time t)

$$t = \frac{2h}{c}$$

What happens if the speed of light is constant?

- In the frame where the experiment is moving, the light appears to follow a triangular path, length $2l$. Time taken for light to complete journey $t' = {}^{2l'}/_{c}$
- Half of the path is a right-angled triangle, so

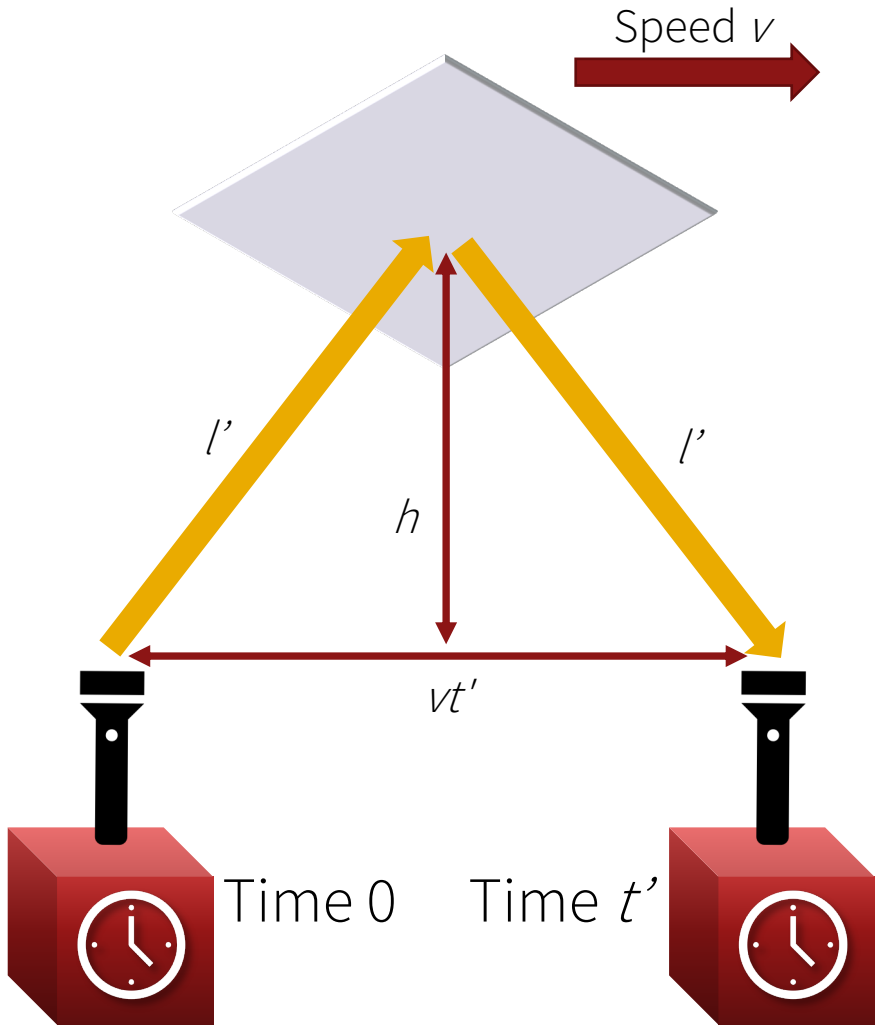
$$l'^2 = \left(\tfrac{1}{2}vt'\right)^2 + h^2$$ TMI

$$(c^2 - v^2)t'^2 = c^2t^2 \text{ (using } l = \tfrac{1}{2}ct)$$

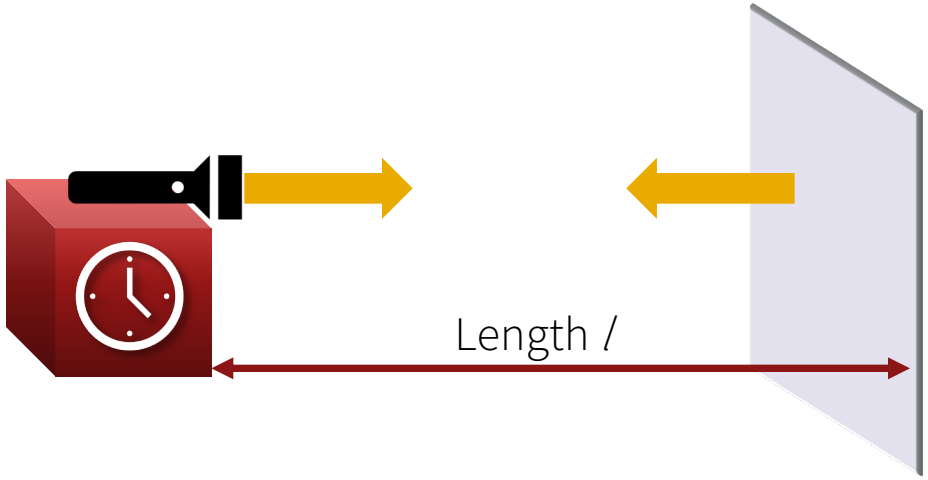
$$t' = \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right)t$$

$$t' = \gamma t$$

Time dilation: the moving clock (t) ticks more slowly than our clock in the 'lab' (t')



What happens if the speed of light is constant?



Time 0

- Send a flash of light to a mirror a distance l away (the length of the experiment) at time 0
- Time how long it takes to come back (at time t)

$$t = \frac{2l}{c}$$

What happens if the speed of light is constant?

- Consider the outward and return journey of the light separately
- On the outward journey, which takes time t_1', while the light is travelling, the mirror has moved away a distance vt_1', so the journey time

$$t_1' = \frac{l' + vt_1'}{c}, \text{ so } t_1' = \frac{l'}{c - v}$$

- On the return journey, which takes t_2', the receiver has moved a distance vt_2' closer

$$t_2' = \frac{l' - vt_2'}{c}, \text{ so } t_1' = \frac{l'}{c + v}$$ TMI

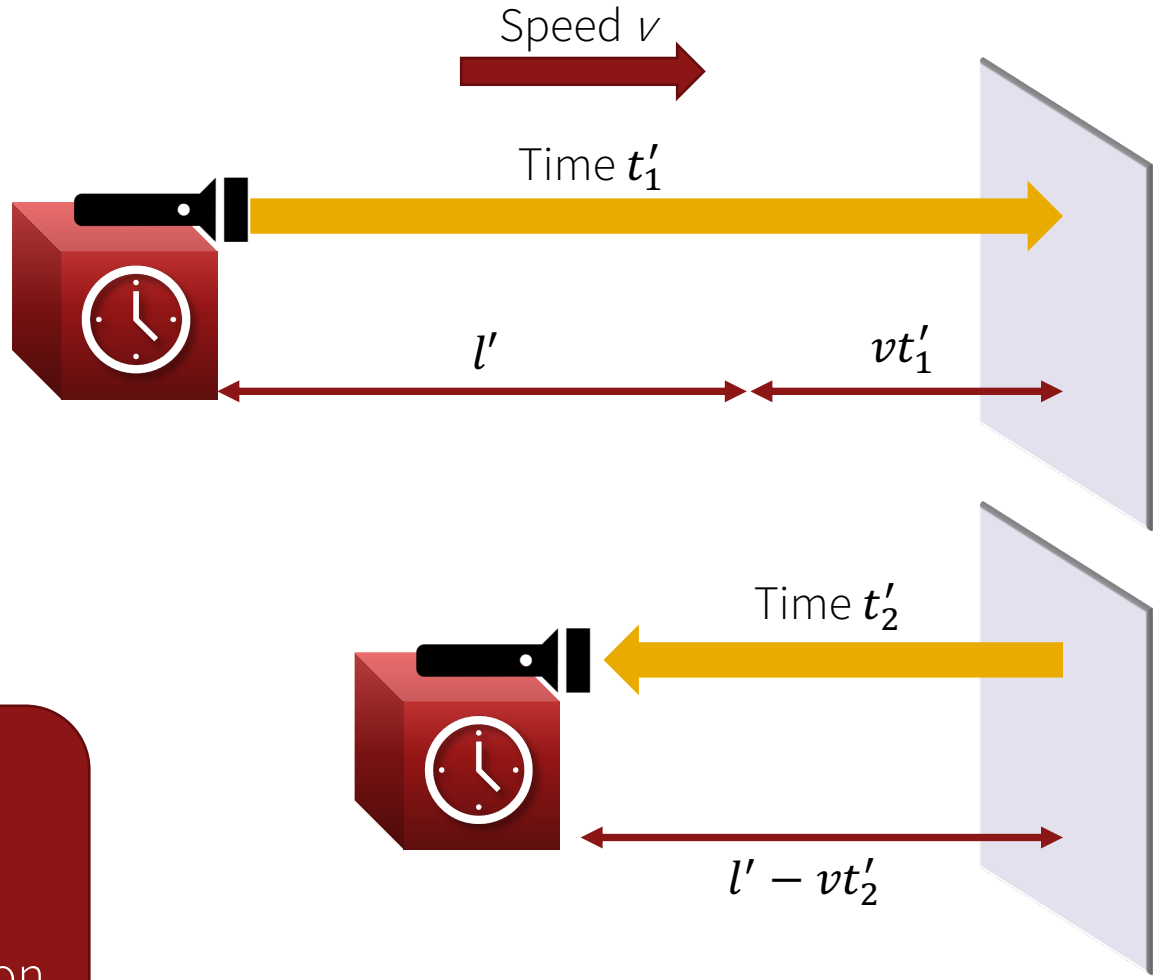
- The total journey time $t' = t_1' + t_2'$, but we know because of time dilation that $t' = \gamma t$, and that $t = {}^{2l}/_{c}$

$$\gamma \frac{2l}{c} = \frac{2l'}{c} \frac{1}{1 - \frac{v^2}{c^2}}$$

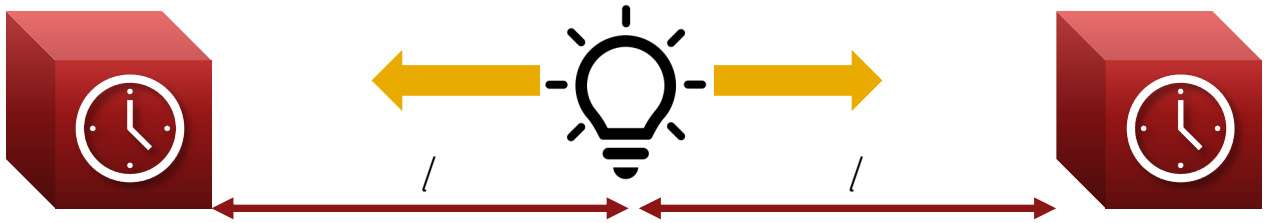
$$= \frac{2\gamma^2 l'}{c}$$

$$l' = {}^{l}/_{\gamma}$$

Length contraction: an object appears shorter (along the direction of travel) when it is moving



When did that happen?



- Send a flash of light towards receivers in opposite directions
- Time how long it takes to arrive at each end (at time t)

$$t = \frac{l}{c}$$

When did that happen?

- When the experiment is moving, the length l is length contracted to $l' = {}^{l}/_{\gamma}$
- The light ray travelling to the left arrives at the receiver at time t_1'. While it is travelling, the receiver has moved a distance vt_1' closer to the source.

$$t_1' = \frac{{}^{l}/_{\gamma} - vt_1'}{c} \text{ so } t_1' = \frac{x}{\gamma c}\left(\frac{1}{1+\frac{v}{c}}\right)$$

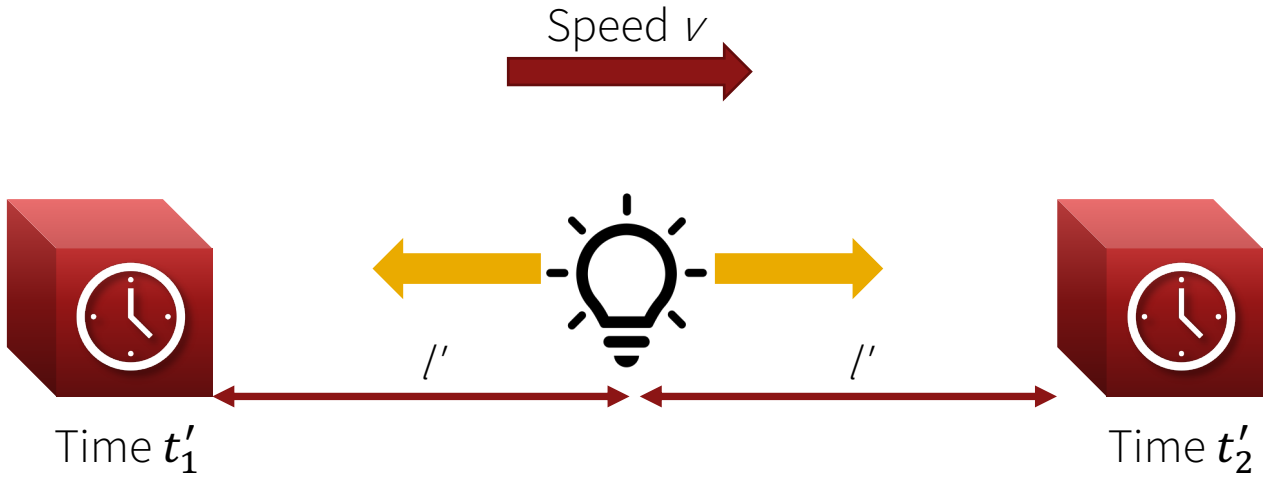
- The light ray travelling right arrives at time t_2' and while it is travelling, the receiver has moved away a distance vt_2'

$$t_2' = \frac{{}^{l}/_{\gamma} + vt_2'}{c} \text{ so } t_2' = \frac{x}{\gamma c}\left(\frac{1}{1-\frac{v}{c}}\right)$$

TMI

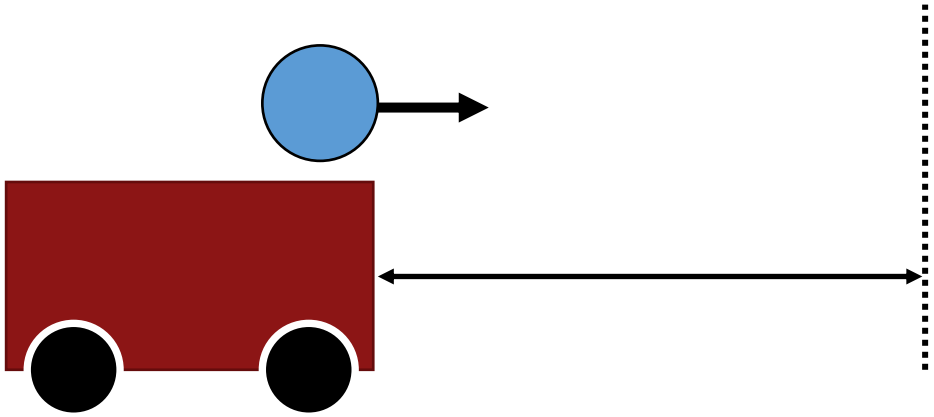
- Putting this together, the time interval between the light rays reaching the two receivers:

$$t_2' - t_1' = \frac{2\gamma v x}{c^2}$$



Even though events may be simultaneous in one inertial frame, they are not simultaneous in another inertial frame (moving at speed v with respect to the first frame), unless the events happen at the same place (x=0)

Adding Velocities



- We need a new velocity addition law to work out problems like:
  - Launching something from a vehicle that is in motion
  - Measuring the velocity of something when you, yourself are in motion
- We can't just use $u' = u + v$ any more
  - If $u = c$, we now require $u' = c$ (constancy of speed of light)
  - If $u \ll c$ and $v \ll c$, we need to recover $u' = u + v$ just as we are familiar with in classical physics

Adding Velocities

- When car is moving, the length l is length-contracted to l/γ, and the distance measure has moved distance vt' with the car
- So, the ball appears to have travelled:

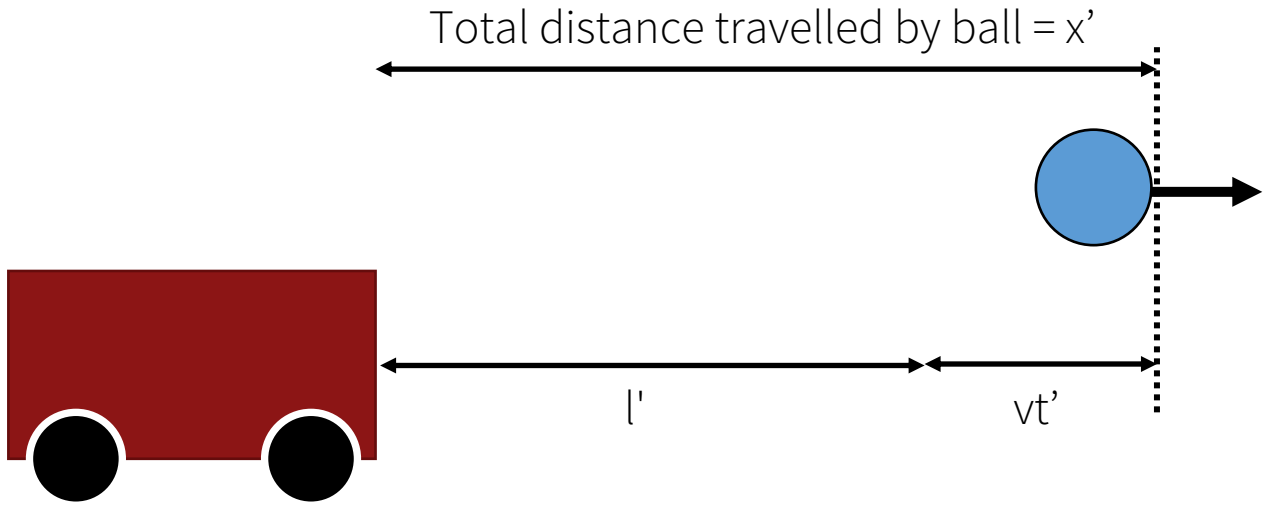
$$x' = \frac{l}{\gamma} + vt'$$

- But we can also measure the distance x' from the car's frame (according to the car, the point on the roadside it launched the ball is moving backwards at speed v, and it is length contracted)

$$\frac{x'}{\gamma} = l + vt$$

- Putting these together:

$$t' = \gamma\left(t + \frac{vl}{c^2}\right) \text{ and } x' = \gamma(l + vt)$$ TMI



Apparent velocity, $u' = x'/t'$

$$u' = \frac{u + v}{1 + \frac{uv}{c^2}}$$

Obey the speed limit!

- Adding velocities (or measuring the velocity u when you're travelling at velocity v):

$$u' = \frac{u + v}{1 + \frac{uv}{c^2}}$$

MaT

- u' is always less than the speed of light, c
- But, we can always travel at any velocity v we like to measure the velocity u. From the principle of relativity, no measurement is more correct than any other

It is impossible to travel faster than the speed of light

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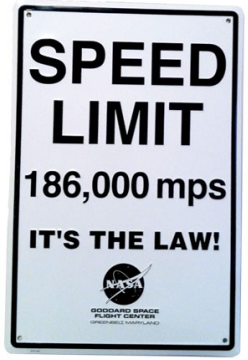

- Time dilation: moving clocks run more slowly
- For a time interval t measured in the lab, the time according to the moving clock t':

$$t' = \gamma t$$

- The Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- If $v = c, t' = \infty$
- If you're travelling at the speed of light, your time stops!

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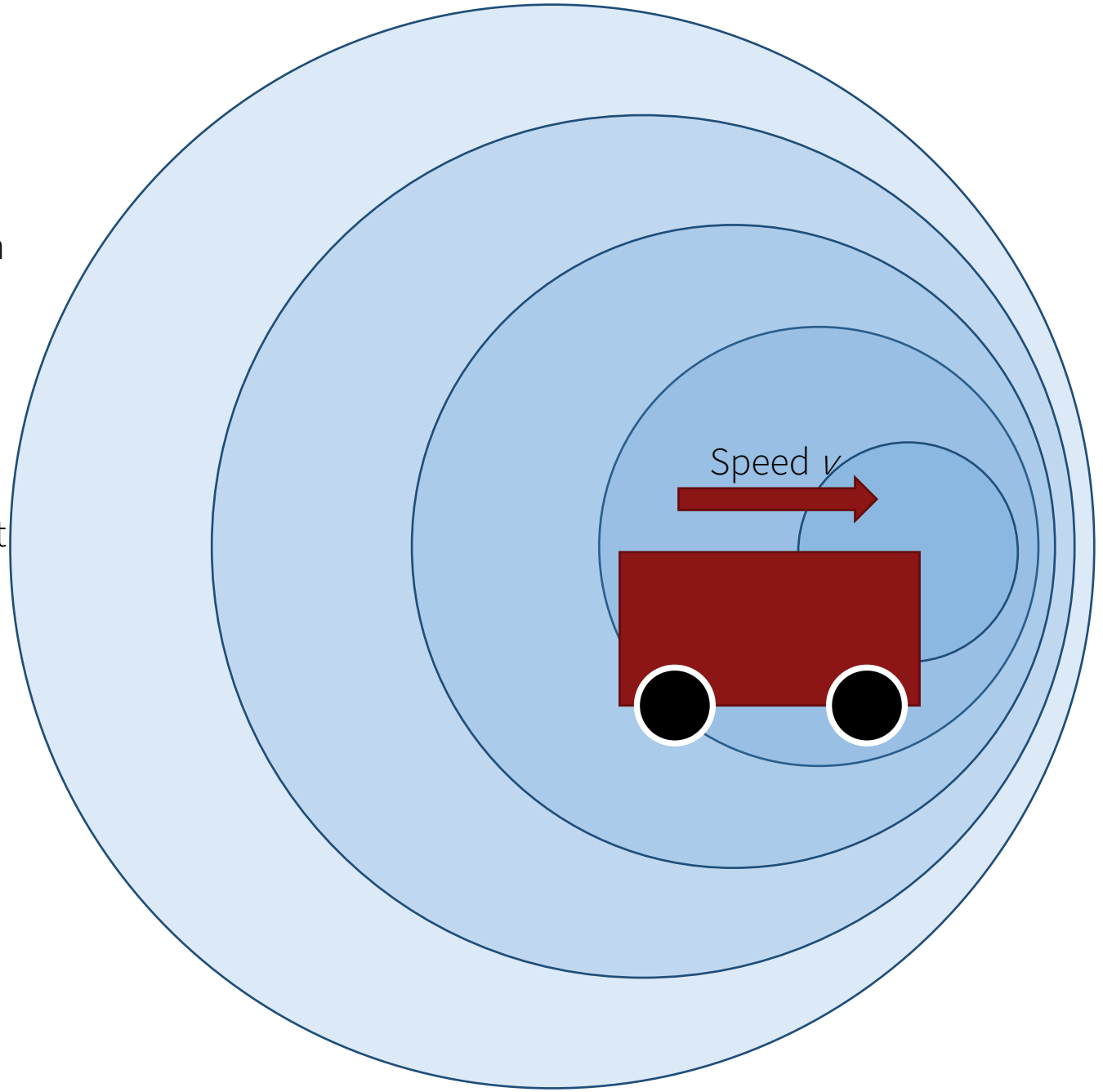
The Doppler Effect

- Distance between the wave crests is the wavelength
- Inverse of the time between crests arriving is the frequency an observer measures (the pitch of sound, or the color of light)
- The siren or light on the vehicle emits wave crests at regular time intervals (as far as it is concerned)
- Due to motion of vehicle, the crests are closer together in front of the motion and further apart behind
  - You observe a higher frequency than is emitted when the emitter is moving towards you
  - You observe a lower frequency than is emitted when the emitter is moving away from you



The Doppler Effect

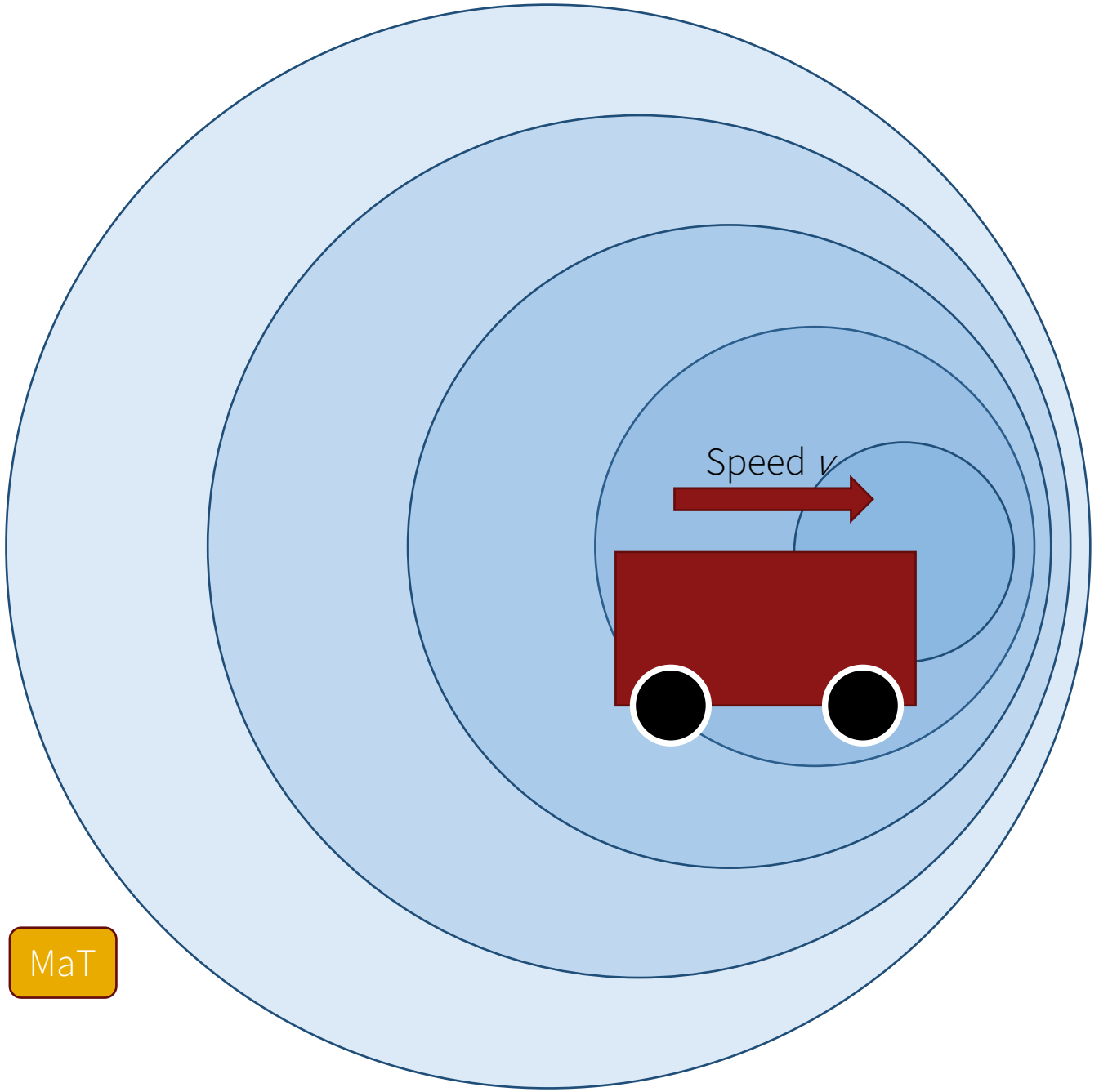
- Vehicle emits wave crests at time interval $\Delta t' = {}^{1}/_{f_E}$
- Because the vehicle is moving, its clock runs slowly compared to ours, so to us, the time interval between emission of wave crests is $\Delta t = \gamma \Delta t' = {}^{\gamma}/_{f_E}$
- The first wave crest is emitted at $t = t' = 0$, it travels distance d at speed c to us, so arrives at time $t_1 = \frac{d}{c}$
- The second wave crest is emitted at time $t = \frac{\gamma}{f_E}$ on our clock, but the vehicle has moved a distance $v\Delta t$ closer to us, so the second crests arrives at time $t_2 = \frac{\gamma}{f_E} + \frac{d - v\Delta t}{c} = \frac{\gamma}{f_E} + \frac{d}{c} - \frac{v\gamma}{cf_E}$
- We measure the frequency from the time interval between the crests arriving

$$\frac{1}{f} = t_2 - t_1 = \frac{\gamma}{f_E} - \frac{v\gamma}{cf_E}$$

TMI

$$\frac{f_{\text{obs}}}{f_{\text{emit}}} = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}$$

MaT



Mass and Energy

- The rest mass, m_0, is the mass of an object measured in its own rest frame
  (i.e. the mass measured when it is at rest)
- Relativistic version of the momentum, $p = \gamma m_0 v$
  - Comparing this to the classical momentum, $p = mv$, we can identify the "apparent mass" $m = \gamma m_0$
- In classical mechanics, the power, $\frac{dE}{dt} = Fv$

WILtY

  - In special relativity, we find that $Fv = \frac{d(\gamma m_0 c^2)}{dt}$
  - Therefore… $E = \gamma m_0 c^2$ or $E = mc^2$

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What does this mean?

- From $E = mc^2$ we can think of mass and energy being equivalent
- Think of mass as another "stored" form of energy
- When an object starts moving, $m = \gamma m_0$, and γ gets larger — we can think of the kinetic energy adding to the mass of the object

As the speed increases, the apparent mass increases, so to accelerate it further, the force we need to apply ($F = ma$) increases.

As $v \to c$, $\gamma \to \infty$ so $m \to \infty$ (unless $m_0 = 0$)

- A massive object can never reach the speed of light, since we'd need to apply an infinite force to accelerate it to c

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Special Relativity in 1 slide

- The speed of light, c (in a vacuum), is constant for all observers, no matter how fast they are travelling
- Length contraction: moving objects appear squashed along the direction of motion: $l = {}^{l'}/_{\gamma}$
- Time dilation: moving clocks tick more slowly: $t = \gamma t'$
- Events that are simultaneous in one frame are not simultaneous in another frame (unless they happen at the same place)
- Nothing can travel faster than the speed of light, c. Only massless particles can travel at the speed of light
- Energy and mass are equivalent: $E = \gamma m_0 c^2$

The Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$\gamma > 1$ and as $\mathbf{v} \to c, \gamma \to \infty$

Q1

A type of subatomic particle called a muon is created when a cosmic ray hits the Earth's upper atmosphere, at an altitude of 15km.

A muon has a lifetime of 2.2μs, but we can detect the muons when they reach the ground.

How is this possible?

Q2

A (very high speed) train is travelling at speed $v = \frac{\sqrt{3}}{2}c$ through a tunnel.

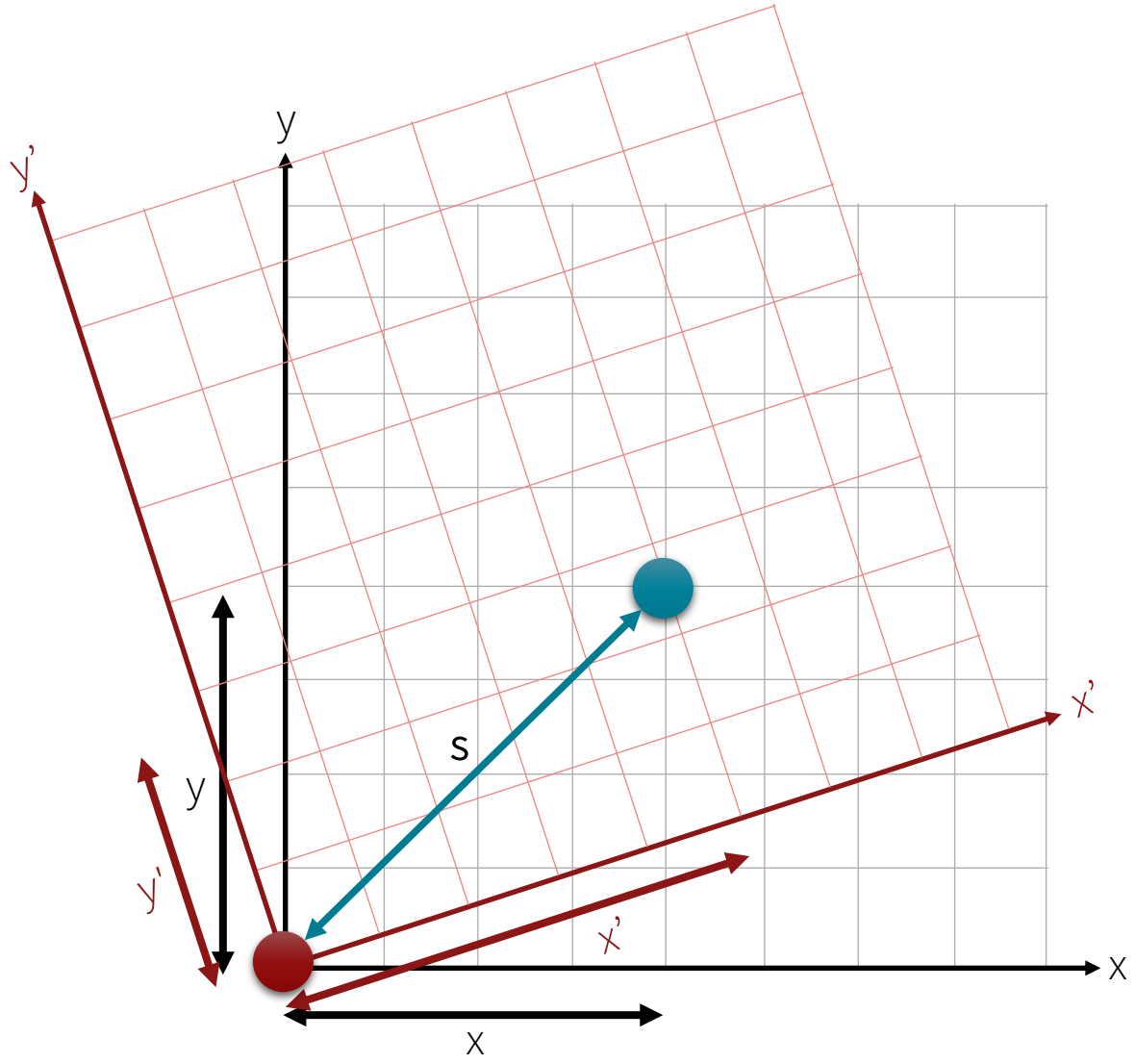
The train is 500m long and the tunnel is 250m long.

Someone is watching from outside. Do they see the train fit into the tunnel?

Someone on the train looks at the tunnel. How long do they think the tunnel is?

How can we reconcile these two observations?

Space and Time



Point P is a distance s from the origin (O)

Can measure its co-ordinates (x, y) on a grid

$$s^2 = x^2 + y^2$$

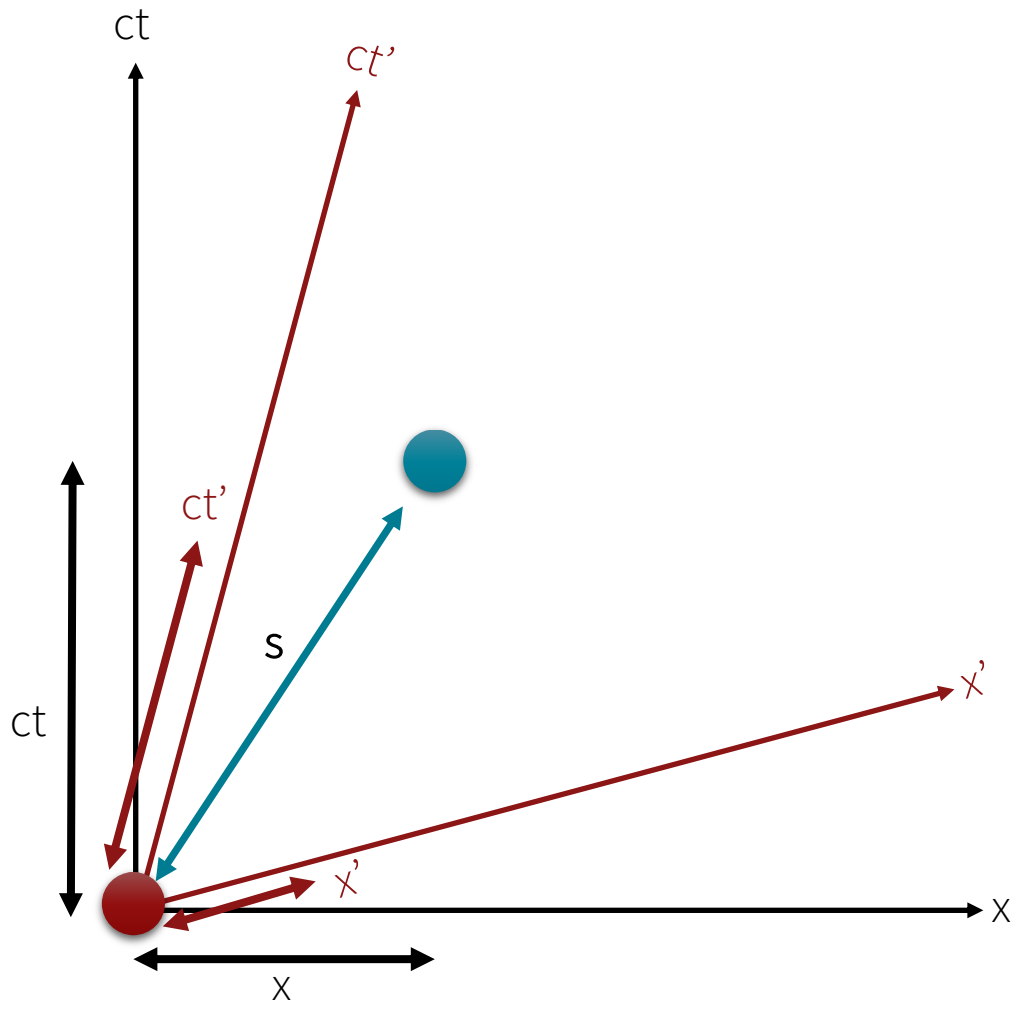
Can draw another, rotated grid and measure its co-ordinates (x', y')

$$s^2 = x'^2 + y'^2$$

It doesn't matter what co-ordinate system you use, the displacement between the two points must always be s

$$x^2 + y^2 = x'^2 + y'^2$$

Space and Time



2 events in 1D space (position x) ant time (t).

The interval:

$$s^2 = (ct)^2 - x^2$$

Lorentz transformation into a moving frame is a rotation

WILtY

The interval (in space + time) between the two events must be the same as measured by all observers

$$s^2 = (ct)^2 - x^2 = (ct')^2 - x'^2$$

Space as seen by one observer is a mixture of space & time as seen by another observer

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Space and time must be unified — spacetime

The – sign between the position and time co-ordinate stops us travelling back in time)

BigI

- The laws of physics are the same in all inertial frames
- The speed of light is constant for all observers
- This means we must re-interpret the notion of space and time — they are no longer absolute
  - Time dilation
  - Length contraction
  - Loss of simultaneity
- The Doppler effect
- Nothing can travel faster than the speed of light. Only massless particles can travel at the speed of light
- Energy and mass are equivalent
- Space and time are unified into spacetime